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Physical interpretation of fracture-toughening mechanisms

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The basic idea behind the toughening of materials by the introduction of energy-absorbing or dissipating artefacts is critically re-examined. It is shown that energy dissipation by plastic deformation or other dissipative processes at the tip of a growing crack does not contribute to increasing the effective surface energy or the crack resistance of the material. Erroneous interpretations of toughening by the presence of fibres or by phase transformations occuring at the tip of a growing crack are discussed. It is argued that all processes which dissipate energy at the crack tip produce crack shielding and that this effect must be an important contribution to toughening. It is concluded that most of the features and properties embodied in methods of toughening can be explained by shielding effects and that the increase in toughness is due to a reduction in the local value of the crack extension force, or its equivalent stress intensity factor, and not to an increase in energy dissipated.

1. Introduction

Linear elastic fracture mechanics (LEFM) has been developed into a successful theory based on the well-founded experimental observation that a crack propagates in most brittle and semi-brittle materials when the crack extension force, $\mathcal{G}$, (also referred to as the energy release rate) reaches a critical value, $\mathcal{G}_c$. This critical value can be regarded as an intrinsic material property which defines the resistance to crack propagation, and when it is expressed in terms of the equivalent stress intensity factor, K_{IC}, is known as the fracture toughness. Experience has also shown that LEFM can be applied to those situations of small-scale yielding in which plastic deformation is confined to a small region around the tip of the crack, and in such cases there is no need to inquire into the physical significance of the critical value of $\mathcal{G}$ nor into the basic instability criterion on which its definition was originally based. Although the original Griffith theory [1] pointed out the significant parameter in the fracture process, namely the crack extension force (or its equivalent stress intensity factor introduced later by Irwin [2]), its critical value must be determined by experiment. For most materials this critical value is found to be much greater than twice the surface energy, 2γ, as originally predicted by Griffith [1], but this is of little practical consequence provided that it is a constant which can be used as a geometry- and load-independent characteristic material property. LEFM can therefore be regarded as a phenomenological theory based on an experimentally verifiable criterion. It has thus been possible to formulate LEFM in terms of the stress intensity factor, K, which is used as a parameter characterizing the deformation at the crack tip, avoiding any possible connection between K and the energy release rate on which the theory was initially based.

Attempts to extend fracture mechanics to more ductile or tougher materials where plastic deformation takes place on a larger scale when a crack grows, have been far less successful. Alternative fracture criteria for crack propagation, such as critical values of the J integral [3, 4] and the crack-tip opening displacement (COD) [4] have been proposed for these materials but ultimately the validity or applicability of such criteria can only be decided by experiment.

In some tough materials, where cracks can be made to propagate in a stable, controlled manner, crack extension cannot be predicted by criteria based on a single parameter. In such materials the resistance to crack propagation seems to increase as the crack grows. The behaviour of the cracks in these materials is usually described by the so-called resistance curves, or R curves.

It is recognized that the development of new tough materials and the explanation of toughening effects has been mostly guided by the intuitive notion that processes which dissipate energy during crack growth contribute to an increase in the toughness of the material. Although this approach appears to work well qualitatively, the principle on which it is based is physically unsound. A particular example of this is the misinterpretation of the role of plastic deformation on crack instability [5–7]. The analogy between plastic work and energy dissipated by other processes during crack growth is used in this paper to discuss some well-known toughening mechanisms. The difficulties encountered with interpretations of toughness based on energy absorbed or dissipated are emphasized and, wherever possible, the way towards a correct formulation and interpretation is pointed out.

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2. The role of plastic deformation on crack instability

The Griffith criterion for crack growth is a thermodynamic instability criterion which proposes that crack propagation takes place in a loaded body, which only deforms elastically, when the crack extension force, $\mathcal{G}$, reaches the critical value $\mathcal{G}_c = 2\gamma$. The function $\mathcal{G}$ can be obtained from elastic calculations of the change in the combined elastic energy of the body and the potential energy of the loading system caused by the extension of the crack alone. For an isotropic elastic body with a through-thickness crack of length c, and a remote tensile stress, σ, normal to the crack plane, the crack extension force can be written as

$$\mathcal{G} = A\sigma^2 c \qquad (1)$$

where A is a constant depending on the geometry of the body and the elastic constants of the material. The instability condition, with respect to crack extension, is thus given by

$$A\sigma^2 c = 2\gamma \qquad (2)$$

Orowan [8] and Irwin [9] independently suggested that the Griffith equation could be modified and applied to materials exhibiting small-scale plastic yielding by adding a plastic work term, p, per unit area of crack surface to the surface energy, γ. This term p is generally taken to be much greater than γ and to have the same sign as γ.

The Orowan and Irwin modification to the Griffith equation is often used in a generalized form in those cases where large plastic deformation and/or other energy-dissipating processes are associated with crack growth. The instability condition for crack propagation is then written as

$$A\sigma^2 c = \mathcal{G}_c = R \qquad (3)$$

where R is said to be a crack resistance term or a rate of energy dissipation, which has to be determined by experiment and which includes all the energy dissipated per unit increment of crack area [10–14].

While Equation 3 can be regarded as a generalization of Griffith's equation, it has also been interpreted as an energy balance based on the principle of energy conservation [12, 13]. Both these formulations of the conditions for crack growth have been discussed elsewhere by the authors [7]. The conclusions, which are drawn from a rigorous analysis of the rôle of plastic deformation on crack instability [6, 7], seem to conflict with the usual interpretation of p and R as crack-resistive terms. A simpler but equally sound physical argument will be given here to clarify this matter.

Consider an elastic body of uniform thickness, t, with a through-thickness crack of length, c, which is loaded with a force P. The body plus the mechanical system applying the load P is considered to be, as implied in Griffith's original formulation, a thermodynamic system which can exchange energy with the environment as heat. The load elongation diagram for this loaded body is represented in Fig. 1. If the length of the crack increases by δc whilst the region of the body where the force P acts remains fixed (the fixed-grips condition) the load drops by δP_c and the elastic energy (a Helmholtz free energy) of the body decreases by δF_c. The total Helmholtz free energy of the body is increased at the same time by $2\gamma t \delta c$ due to the formation of extra crack surfaces. If, at the same time, plastic deformation takes place (near the crack, or elsewhere), the load drops by an extra amount δP_p due to the conversion of elastic strain (or elongation) into permanent plastic strain (or elongation δl_p). It is clear that the free-energy change of the system caused by the plastic deformation is a negative quantity since, like crack growth, it reduces the strain energy of the body, in this case by an amount δF_p. This is the energy extracted from the body to produce the plastic deformation and is therefore the plastic work. From a formal point of view, and as far as computing energy changes is concerned, no distinction needs to be made between the plastic deformation which takes place at the crack tip and may be induced by the presence of the crack, and that which may occur elsewhere in the material.

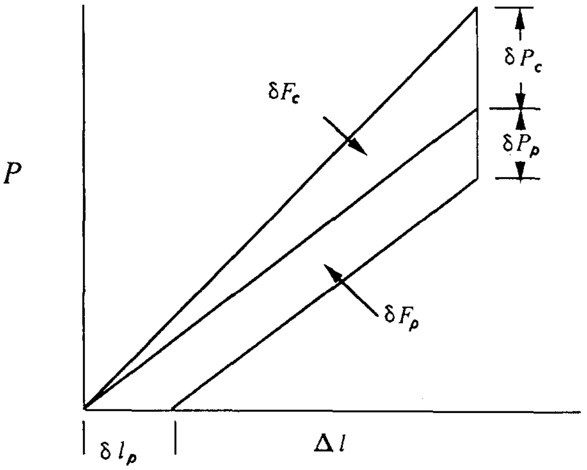

Figure 1

Of course the result would be the same if crack extension had taken place under fixed load rather than under fixed grips, and, in general, plastic deformation, like crack growth, will always reduce the combined elastic energy of the body plus the potential energy of the loading system, added with due attention to sign. Plastic work is not therefore energy which is stored in the sample, but is the free energy, δF_p, extracted from the body and/or loading system to produce the plastic deformation. Most of this energy is dissipated as heat in the environment (if the environment acts as a constant-temperature sink), and only a small fraction, α, is stored in the material as strain energy associated with dislocations and other defects produced by the plastic deformation. It is essential to note that for the formulation of a criterion of instability with respect to crack extension, as in Griffith's original theory, only the free-energy changes in the system as defined above need to be taken into account; the energy changes in the surrounding environment are not counted. Therefore the energy dissipated as heat in the environment (the largest part of the plastic work) represents a negative energy change. The sum of all the free-energy changes associated with both crack growth and plastic deformation gives the total free-energy change, δF_p, in the system as defined above, so that

$$\delta F_T = \delta F_c + \delta F_p + 2t\gamma\delta c + \alpha\delta F_p \qquad (4)$$

and

$$\frac{1}{t}\left[\frac{\partial F_T}{\partial c}\right]_{T,l} = \frac{1}{t}\left[\frac{\partial F_c}{\partial c}\right]_{T,l} + \frac{1}{t}\left[\frac{\partial F_p}{\partial c}\right]_{T,l} + \alpha\frac{1}{t}\left[\frac{\partial F_p}{\partial c}\right]_{T,l} + 2\gamma \qquad (5)$$

where the sub-indices indicate changes at constant temperature, T, and length, l. Here the terms $(\partial F_c/\partial c)_{T,l}$ and $(\partial F_p/\partial c)_{T,l}$ are both negative energy changes whilst the terms 2γ and $\alpha(\partial F_p/\partial c)_{T,l}$ are positive because they both increase the free energy of the body. The total free-energy change, δF_T, will in general be different from zero because Equation 4 is not a statement of energy conservation. It will only be zero if the system is in equilibrium (stable or unstable) with respect to crack extension. Of course if $\delta F_p = 0$, the Griffith equation is obtained from Equation 5 by setting $(\partial F_T/\partial c)_{T,l}$ to zero, but if δF_p is different from zero a plastic work term appears with a negative sign in Equation 5. The addition of a plastic work term to the surface energy, 2γ with a positive sign is therefore unjustified. Even in the most unthinkable case where $\alpha = 1$ (when all the work of plastic deformation is stored as elastic energy in the body) there is no positive plastic work term to be added to the surface energy and the Griffith equation would be unmodified. The consequence is that if the plastic work term $(1 - \alpha)(\partial F_p/\partial c)_{T,l}$ which is a negative quantity, is included in Equation 5 to determine a crack instability condition, then plastic deformation, far from opposing crack growth would provide an increased crack driving force. This apparent paradox does not arise if the free-energy changes resulting from plastic deformation are not included in Equation 5. A rigorous analysis of this problem shows [6] that in fact crack growth and plastic deformation are independent, though interacting, processes which do not compete for a limited supply of energy but draw energy independently from the same sources. The crack extension force, $\mathcal{G}$, given by the left hand side of Equation 1, or its equivalent stress intensity factor, K, obtained from elastic calculations, is derived from the rate of energy change $(\partial F_c/\partial c)_{T,l}$ produced by crack extension alone, although it may be affected by the presence of dislocations.

The energy criterion used in LEFM is based on the critical value of $\mathcal{G}$, or K. Alternative, and different, definitions of toughness have also been proposed based on the total energy changes associated with both crack extension and plastic deformation (the terms $(\partial F_c/\partial c)_{T,l} + (\partial F_p/\partial c)_{T,l}$ in Equation 5) [15, 16]. Although such definitions of fracture toughness have little physical justification [6], they may still be acceptable for practical purposes if experiment could prove that they are geometry- and load-independent material properties.

It is concluded that the effects of plastic deformation on crack propagation cannot be adequately explained by a generalization of the Griffith criterion which includes plastic work as a resistive term.

A similar criticism can be raised against interpretations of toughening mechanisms based on irreversible processes which occur during crack extension. The analogy between energy dissipated by plastic deformation and energy dissipated by other mechanisms or processes is clearly evident since both represent energy extracted from the system and dissipated into the surrounding environment. In the following sections a discussion of some well-known accepted toughening mechanisms is given.

3. Toughening of a matrix by long fibres

On the basis of experimental observations, the failure of a fibre-reinforced material and the contribution of the fibres to the toughness of the material is usually modelled by the following series of events considered with reference to Fig. 2 which represents part of a fibre-reinforced sample of length l loaded in the fixed-grips condition by a force P at its ends in a direction parallel to the fibres [17–22]. It is supposed that a crack grows in the matrix between the fibres, its surface area increasing by δa_m. This will cause the load to drop by δP_m and some elastic strain energy, δF_m, will be released from the sample as the stress generally relaxes. At the same time, however, the load in the unbroken fibres crossing the crack increases. If the fibres are rigidly bonded to the matrix and no sliding between fibres and matrix is allowed, the relaxation of the crack surfaces will be small with a correspondingly small change, δF_m, in free energy and a very small driving force for the crack to open. If the fibres have a high Young's modulus and they are not allowed to slide, both matrix and fibres will break together, as in a homogeneous body. The only increase in toughness which the fibres can provide in this case must come from the toughness of the fibres themselves. This "no-sliding condition" will not be examined further because it is a particular case of the more important situation where some de-bonding and sliding of the

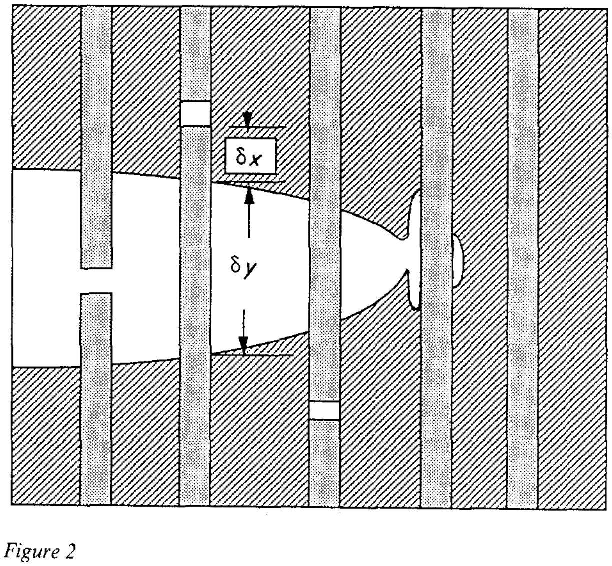

Figure 2

fibres is allowed over a length δy. This length is determined by the distance over which the shear stress along the fibre–matrix interface exceeds a certain sliding friction stress, τ_f, which is assumed to be non-zero. If the crack opens in this manner in the fixed-grips condition then the force, P, on the sample will drop further and the elastic energy (a Helmholtz free energy) will be decreased by an amount δF_f. When the elastically stretched fibres eventually break, creating a fibre fracture surface of area δa_f, extra elastic energy δF_d is released not only from the broken fibres as is often assumed but also from the whole sample. After failure of a fibre there may be some length δx of broken fibre which slides out of the surrounding matrix (see Fig. 2), and this produces further relaxation of the stress and further decrease of the elastic strain energy by δF_p.

The total decrease (or negative change) in the elastic strain energy of the sample is $\delta F_m + \delta F_f + \delta F_d + \delta F_p$ and this provides the driving force for the sequence of cracking and sliding events. On the other hand the free energy of the sample will have increased due to the formation of the matrix crack free surfaces, $2\delta a_m$, the fibre crack free surfaces, $2\delta a_f$, and the debonded fibre–matrix interface, δa_{fm}. This free energy increase is given by

$$2\gamma_m \delta a_m + 2\gamma_f \delta a_f + (\gamma_f + \gamma_m - 2\gamma_{fm})\delta a_{fm}$$

where γ_f and γ_m are the surface energies of fibre and matrix respectively, γ_{fm} is the surface energy of the fibre–matrix interface and δa_{fm} is the area of the debonded interface.

The total free energy change, δF_T of the composite sample is therefore given by

$$\begin{aligned}\delta F_T &= \delta F_m + \delta F_f + \delta F_d + \delta F_p + 2\gamma_m \delta a_m \\ &\quad + 2\gamma_f \delta a_f + (\gamma_f + \gamma_m - 2\gamma_{fm})\delta a_{fm} \quad (6)\end{aligned}$$

Again it is noted that the sum of all the free-energy changes is not in general equal to zero. It is zero only if a condition of instability with respect to crack extension exists for this system. In Equation 6 only the surface energy terms are positive because they contribute to the increase in the energy of the sample due to the formation of extra free surfaces. The other terms are all negative because they represent decreases in the strain energy of the sample.

In all the conventional models of fibre toughening based on energy considerations great emphasis and importance is placed on the work done against frictional forces during sliding or fibre pull-out. This work is calculated as a separate quantity and is then included in the evaluation of the energy changes with a positive sign [17–22]. It must be clear from our formulation and discussion that the work of fibre sliding and fibre pull-out are the quantities δF_f and δF_p which are both negative, as was the work of plastic deformation in the case where plastic deformation accompanies crack growth.

The fact that fibre sliding or pull-out is not frictionless and that some, or all, of the work of friction is dissipated as heat to the environment is irrelevant and of no consequence as far as the formulation of the instability criterion is concerned. Only the difference between the final and initial energies of the system needs to be considered; the energy gained by the surrounding environment does not enter the equation.

Because our formulation is at variance with the accepted energy-balance models it is perhaps worthwhile to repeat the argument for the case where the crack extension takes place under constant force.

When the crack grows in the matrix by δa_m and there is no sliding of the fibres, the sample length increases by a small amount δl_m and if the fibre slides over a distance δy there is a further increase δl_f in the length of the sample. The elastic strain energy of the sample (a Helmholtz free energy) increases by $\delta F_f + \delta F_m$ and the work done by the constant force, P, is $P(\delta l_f + \delta l_m)$.

When the fibres break and pull out of the matrix, there is a subsequent increase in sample length, δl_d and δl_p, due to fibre rupture and pull-out, respectively. The constant force does work equal to $P(\delta l_d + \delta l_p)$ and the increase in strain energy of the sample is $\delta F_d + \delta F_p$. It is to be noted that if the deformation of the matrix and fibres is linear elastic then the increase in strain energy is equal in magnitude to half the work done by the external load, irrespective of how the stresses are redistributed between matrix and fibres. Existing calculations of the energy changes based on a local redistribution of stress, yield results which do not satisfy this condition and cannot therefore be correct [17, 21, 22].

The total change in the combined strain energy of the sample and potential energy of the loading system (the Gibbs free energy) is therefore

$$\begin{aligned}\delta G &= (\delta F_m + \delta F_f) - P(\delta l_m + \delta l_f) \\ &\quad + (\delta F_d + \delta F_p) - P(\delta l_d + \delta l_p) + 2\gamma_m \delta a_m \\ &\quad + 2\gamma_f \delta a_f + (\gamma_f + \gamma_m - 2\gamma_{fm})\delta a_{fm} \\ &= -\delta F_m - \delta F_f - \delta F_d - \delta F_p + 2\gamma \delta a_m \\ &\quad + 2\gamma_f \delta a_f + (\gamma_f + \gamma_m - 2\gamma_{fm})\delta a_{fm} \qquad (7)\end{aligned}$$

and this is the same as Equation 6 because the terms δF_m, δF_f and δF_p are now positive quantities which appear with a negative sign. As before in Equation 6, the work of fibre sliding and fibre pull-out is included in these terms. This work is not energy absorbed by, or stored in, the sample and it is therefore incorrect to include any of these work terms with a positive sign in the total energy changes to make them appear as an increased surface energy or as crack-resistive terms.

Some existing models of fibre toughening in which the energy dissipated acts to increase the crack resistance, the effective surface energy, or the work of fracture, are based on the notion that energy dissipated is unavailable to crack growth [19, 20]. This is evidently inspired by an energy conservation criterion which is always satisfied whatever the crack length, irrespective of whether the crack grows or not, and therefore it cannot provide a criterion for crack extension on its own [7].

The instability condition with respect to crack extension is obtained by setting $(\partial F_T/\partial a_m)_{T,l} = (\partial G_T/\partial a_m)_{T,P} = 0$ and it can be seen that the inclusion of the work terms in the energy changes used to formulate a matrix crack instability criterion would produce an increased crack driving force rather than an increase in crack resistance. This paradox is similar to that encountered in Section 2 when discussing the effect of plastic deformation on crack growth and it highlights the fundamental difficulty in defining a criterion based on a global energy balance for crack instability in a composite material. This is because crack growth in these complex materials takes place by a series of sequential events such as matrix cracking, debonding and fibre rupture. It should be possible, in principle, to define in this case a crack extension force for each one of these events separately but it is doubtful whether a meaningful definition of a global crack extension force, encompassing all the events together, is possible. A more complete discussion of this point in connection with the analogous case of plastic deformation is given elsewhere [6].

Further confusion is created by the use of the word toughness as the "total fracture energy" obtained from the area under the force elongation curve up to complete failure divided by the apparent area of the fracture surface. This "total fracture energy" is a definition of toughness entirely different from that based on the critical value of the crack extension force or critical value of K. It is not even equal to the rate of energy dissipation of Equation 3. The indiscriminate use of, and conversion between, these different kinds of toughness is often made in the literature and this is totally unjustifiable [19, 23].

4. Phase-transformation toughening

This refers to the increase in toughness observed to occur in some steels and in some ceramic materials containing ZrO_2 particles when a stress-induced phase transformation in a particulate second phase occurs in the stress field of the growing crack. Some early explanations of this toughening effect were based on the fallacious argument that "the phase transformation in front of a growing crack is equivalent to a non-elastic deformation, and as such, is capable of absorbing energy that would otherwise be available for crack extension" [24], or that "the extra toughness available can be estimated from the release of strain energy needed to compensate for the changes in free energy of the transformed particles" [25]. For a phase transformation to occur at all in a loaded sample, the free energy of the whole system must remain unchanged (if the system is in equilibrium) or decrease. From this simple consideration it should be clear that the phase transformation does not "absorb" energy in the sense that it does not increase the free energy of the system, but decreases it as does plastic deformation.

The free-energy changes associated with a phase transformation which takes place in a sample loaded at constant force can be obtained in different ways, but basically the following need to be considered. There is a change in the bulk free energy of the transforming particles, δF_0, which is the difference in their Helmholtz free energies in the transformed and untransformed states when the particles are stress-free. This is a negative quantity which provides the driving force for the transformation. If the transformation results in volume and/or shape changes, there are further changes δF_p and δF_m due to the change in strain energy of the transformed particles and to their misfit strain energy in the matrix, respectively. Both these changes are positive and oppose the transformation. The work, δW_t, done by the external load during the transformation can be positive or negative depending on whether this opposes or assists the transformation. In the cases of interest in transformation toughening, where the phase change of the particles takes place near a crack, an extra free energy term, δF_i, arises due to the interaction between the stress field of the crack and the misfit fields of the particles. Since the transformation is induced by the presence of the crack it is to be understood that this term is intrinsically negative, as it must increase the driving force for the transformation. Particles in regions where the contribution to the magnitude of δF_i is greatest will be those more readily transformable.

The total Gibbs free-energy change due to the transformation of particles near to a crack is therefore

$$\delta G_t = \delta F_0 + \delta F_p + \delta F_m + \delta F_i - \delta W_t \quad (8)$$

Further changes due to differences in particle/matrix interfacial energies before and after transformation and due to changes in the modulus of the transformed particles could be added to this sum but will be neglected here because they do not affect the argument. It is far more important to note that for the transformation to be possible at all δG_t must be a large negative quantity since in practice martensitic transformations require large driving forces.

Consider now the Gibbs free energy change, δG_c, due to the extension of the crack. The crack surface area increases by δa, the strain energy increases by δF_c, the load does extra work δW_c equal to twice δF_c and

$$\delta G_c = \delta F_c - \delta W_c + 2\gamma\delta a = -\delta F_c + 2\gamma\delta a \quad (9)$$

where γ is the surface energy.

There are additional free energy changes occurring as the crack extends which have not yet been considered. As the crack grows some particles which may have transformed in regions ahead of the crack or at the crack tip will be transferred to the flanks of the crack, and the contributions of these particles to both δF_m and δF_i will change. The strain energy in the particle misfit field, (the particle's contribution to δF_m), will decrease because of the proximity of the traction-free crack surfaces, while the contribution to the interaction energy term, δF_i, increases (from a negative value) as the stresses of the crack field are reduced to near zero behind the crack tip. It is here assumed that no reverse transformation takes place in the crack flanks; if this did happen the Gibbs free energy would be reduced even further.

It is difficult to predict how large these changes in δF_i and δF_m will be, but they are of opposite sign and the net result cannot yield a positive δG_t overall. The sum of all the Gibbs free energy changes which take place when the crack extends and some particles transform simultaneously gives the total Gibbs free-energy change,

$$\delta G = -\delta F_c + \delta G_t + 2\gamma \delta a \qquad (10)$$

In some models of transformation toughening the terms in Equation 8 are carefully and adequately considered [25], but there has been a failure to realize that δG_t is a negative quantity, otherwise no phase transformation would be possible [24–26]. It is therefore incorrect to include this term or some of the negative terms in Equation 8 with a positive sign in the sum of the free-energy changes in Equation 10 to make them appear as an increased effective surface energy or as a crack-resistive term and thus explain the increase in fracture toughness [24–26]. Again, if the free-energy changes associated with the phase transformation are included, with their correct sign, in the computation of the free-energy change used to obtain a global instability criterion from the condition $(\partial G/\partial a)_{T,P} = 0$, they will provide an increased crack driving force.

As in the case of plastic deformation, the energy changes associated with the phase transformation should not be included in the evaluation of the crack extension force.

It seems to be generally accepted that particles transforming ahead of the crack tip make a negligible contribution to toughening and that the largest contribution arises from the "residual energy density stored in the wake of the crack" [27–29]. While it is true that the energy of interaction, δF_i, between the crack stress field and the stress field of the particles increases when these translate behind the crack, this is only part of the total free-energy change when transformation plus crack growth occurs. It would therefore be incorrect to take this positive contribution to the energy changes as the term responsible for the increase in toughness in the context of an overall energy-balance criterion. Furthermore, it is erroneously argued that the energy density stored in the transformation zone originates in the hysteretic behaviour of the region of material containing the transforming particles as it undergoes a loading–unloading cycle [28, 29]. Similar erroneous conclusions are drawn for other toughening mechanisms based on energy "deposited" at the crack flanks [30–33]. This has probably originated from a wrong interpretation of the J integral and its application. (For a more complete discussion of this question see [34].)

5. Discussion

In a discussion of existing interpretations of toughening it is important to clarify that different definitions of fracture toughness have been used, often indiscriminately, in the literature. One is based on a thermodynamic criterion for crack instability as in Griffith's original work, in which fracture toughness is the value of the critical parameter, $\mathscr{G}_c$, or K_c, which is obtained experimentally using elastic calculations of the energy changes associated with crack extension alone. We have argued here and elsewhere [6, 7] that this critical value is not the rate of energy dissipation by irreversible processes taking place during crack growth.

The rate of change, with respect to crack area, of the total energy dissipated by all the processes which take place at the same time as the crack grows, is a different definition of toughness which is not based on a fracture instability criterion. It would therefore be difficult to use it to make predictions of the behaviour of a material as if it were a characteristic material property. However, fracture toughness or fracture resistance thus defined does certainly scale up with the rate of energy dissipation and it constitutes the basis for some erroneous interpretations of crack resistance curves [11].

Since it is obvious that both plastic deformation and other processes which dissipate energy contribute to an increase in the resistance to crack propagation it must be the rôle of fracture mechanics to explain this experimental fact in a consistent and physically acceptable manner compatible with the laws of thermodynamics.

Plastic deformation and other energy-dissipating processes surely affect the propagation of cracks in many different and complex ways, and one of the important effects must be the relaxation, or reduction of stress, which they produce at the tip of the crack. This effect which has been referred to as the "shielding" of the crack from a remote stress has been recognized and considered by many research workers in terms of the reduction of the stress intensity factor K [35, 36]. Shielding, or stress reduction at the crack tip, is an unavoidable consequence of any mechanism such as plastic deformation which dissipates energy. This stress reduction lowers the local value of both the crack extension force $\mathscr{G}$, and the stress intensity factor, K, so that the critical condition for crack instability is more difficult to achieve.

The effects of shielding are more intuitively obvious when expressed in terms of the stress intensity factor, but it is important to realize that explanations based on the local value of the crack extension force are entirely equivalent [6].

In transformation-toughened materials the gradient of the interaction energy between the stress field of the transforming particles and the stress field of the crack produces interaction forces which have a net shielding effect. Some calculations of this effect purport to show that phase changes which produce only a volume change in regions directly ahead of the crack do not cause any shielding [27], provided that the crack tip remains outside this region, but a large shielding contribution appears as soon as the crack attempts to penetrate the transformation zone. Since the transformed zone must always be at the crack tip and translates along with it, there will always be a shielding effect. Further shielding may arise as a result of the large shear components of the martensitic phase change in ZrO_2 particles, although the effect of this may be reduced by twinning.

In some materials, mainly ceramics, a zone of micro-cracks is often formed at the tip of the main crack. This is attributed to the presence of residual stresses and their interaction with the field of the main crack [29], or, in transformation-toughened ceramics, to the strains produced by the tetragonal-tomonoclinic transformation of ZrO_2 particles [37]. The formation and presence of this micro-cracked zone is likely to have an influence on the toughness of the material, but it is by no means obvious whether this should result in an increase or a decrease in fracture toughness. The formation of the micro-cracks and their interaction with the main crack is a complex process involving several steps. The nucleation and growth of the micro-cracks themselves must satisfy an energy criterion, but when the micro-cracks are formed they will both shield each other and shield the main crack which could then propagate by the rupture of ligaments between micro-cracks. This is a case analogous to that discussed in connection with fibre toughening in Section 4 and it should therefore be possible, in principle, to define the crack extension forces for each individual event involved in the propagation of fracture. It is however far less clear whether it is possible to define a global crack extension force based on an overall instability criterion for the growth of the main crack, assuming that it is possible to identify one as such.

In fibre-reinforced materials where the fibres remain unbroken as a crack grows around them in the matrix, the extra load taken by the fibres both at the crack tip and behind the crack front, helps to reduce the stress at the tip of the crack and the magnitude of the crack extension force. In these materials the effects of shielding must be important, and can be modelled by a distribution of point forces, or even a continuous distribution of force, on the crack surface acting in such a direction as to tend to close the crack. The magnitude of these forces depends on the elastic strain of the segments of unbroken fibres and this in turn depends on whether the fibres are rigidly bonded to the matrix, or whether sliding between the matrix and the fibre is allowed. Those fibres which cannot debond nor slide, will carry a higher stress, and will be those more likely to break so that fibres that can slide and remain unbroken will produce the most effective shielding. A sound model of fibre toughening based on this concept of crack bridging has been developed in considerable detail [22, 38–40], and is a good example of how all the features of fibre toughening can be accommodated into models which do not involve erroneous arguments based on the work of fibre pull-out, or work of friction, and energy dissipation.

Some explanations of the R-curve behaviour of alumina [44] have been based also on the misunderstanding and misuse of the fracture energy concepts discussed in this paper. The increase in toughness and the development of R-curve behaviour which arise from the bridging of the crack faces in coarse grained alumina have been attributed [41] to the energy dissipated by the frictional processes involved in the bending, rotation and sliding of the bridging ligaments as the crack opens. It has been explained here that the energy dissipated (per se) does not contribute to an increase in fracture toughness, and to equate the increase in toughness to the area of the load-deflection hysteresis loop generated by the frictional processes is clearly wrong. The effect of crack bridging on the toughness of coarse grained ceramics can be properly explained, like fibre toughening, in terms of the crack tip shielding produced by closure forces on the crack faces [42, 43].

The toughening of polymeric matrices by rubber particles [44] can be discussed in the same way as fibre toughening. Apart from any stress relaxation due to plastic yield of the matrix at the crack tip, further stress relaxations result from the deformation of the softer, or lower modulus, rubber particles and from the bridging and stretching of those particles that, as in fibre-reinforced materials, are left behind the crack front and restrict the crack opening. All these shielding effects would produce a reduction in the crack extension force of the matrix crack which would result in an apparent increase in toughness. It is not suggested here that shielding alone can provide a complete explanation of toughening; this is perhaps a matter for further investigation. It is clear, however, that descriptions based on energy dissipation are fallacious and they cannot provide a valid explanation for the increase in toughness. On the other hand, shielding can explain most of the features of toughened materials, the increases in toughness arising from a reduction in the local value of the crack extension force and not from an increase in energy dissipated.

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